GAS VELOCITY VARIATION IN IONIZING SHOCK WAVES THE PROBLEM OF THE CONDUCTIVE PISTON

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Results of investigation of velocity variation in ionizing shock waves arbitrarily oriented with respect to a magnetic field, and propagating in a gas the magnetic viscosity of which is considerably greater than the remaining dissipative coefficients are presented. A two-dimensional case is considered, i.e. it is assumed that the gas velocity lies in a plane drawn through a normal to the ionizing wave and the magnetic field upstream of it. The results are used for analyzing the solution of the self-similar problem of the conductive piston.

We shall adduce the results of investigation of velocity variation in ionizing shock waves obtained by the analysis of the shock wave structure. We denote by u and v changes of the x- and y-velocity components in a system of coordinates in which the x-axis is selected along a normal to the wave, and $H_{y1} \geq 0$, $H_{z1} = 0$, where $\mathbf{H}_1$ is the magnetic field upstream of the wave. We shall assume the wave to be two-dimensional, i.e. downstream of the wave $H_z = 0$.

In the case of high magnetic viscosity considered below there may exist five kinds of ionizing waves [1]. Certain kinds of such waves were also considered in papers [2 and 3]. A fast wave is characterized by inequality $a_A \leqslant u_2 \leqslant a_+$, and in the case here considered can only be a supersonic one $a_0 < u_1$ (u_1, u_2 are the gas velocities relative to the ionizing wave, in front and behind the latter, a_0 is the gas-dynamical velocity of sound in front of the ionizing wave, and a_+, a_A are respectively a fast magneto-sonic wave, and the Alfven velocity behind the wave). As in this wave the magnetic field does not change [1 to 3], the variation of the gas velocity tangent component is zero. In the case shown on the diagram (Fig. 1) points of the u-axis lying to the right of point E correspond in the uv-plane to a fast shock wave.

An intermediate wave is characterized by inequalities $a_- \leqslant u_2 \leqslant a_A$ (a_- is a slow magneto-sonic velocity). In the two-dimensional case here considered there exist intermediate waves in which the magnetic field does not change, as well as intermediate waves in which the tangential component of the magnetic field H_y undergoes well-defined variations and changes its sign. If the intermediate wave is subsonic $u_1 < a_0$, then in front of it must be fulfilled an additional condition which stipulates that the parameters of state ρ_1, T_1 of the gas upstream of the ionizing wave must be critical, i.e. they must belong to curve $\Phi(\rho, T) = 0$ which in the ρT-plane separates areas of $\sigma = 0$ and $\sigma > 0$. This condition defines the intensity of the gas-dynamical shock wave which propagates throughout the initial state, and ensures the fulfilment of this condition upstream of the ionizing wave. Intermediate subsonic waves always alter the magnitude and the sign of the magnetic field tangential component. On the diagram points lying along segments BC, DE, PG correspond to velocity variations in intermediate supersonic waves.

In a slow shock wave defined by inequality $u_2 < a_-$ the magnetic field tangential component varies arbitrarily within certain limits. Increments of the two velocity components vary in slow supersonic ionizing waves ($u_1 > a_0$) arbitrarily within certain limits, hence, in the uv-plane certain areas correspond to slow shock waves. On the diagram this area is bounded by the curvilinear polygonal curve $NCDFGKL$. Its boundary consists of segments of curves along which one of the following conditions is fulfilled.

1. The ionizing wave velocity is equal to the velocity of the gas-dynamical shock wave behind which the state of gas becomes critical (curve NL).

2. $u_2 = a_{-2}$ (curves CD and LKG). Slow ionizing waves which satisfy condition $u_2 = a_{-2}$ are called Jouguet waves [3 and 4].

3. Velocity variations in a slow ionizing wave is equal to velocity variation in the intermediate and the slow magnetohydrodynamic waves which follow one another at equal velocities (curves NC and DFG).

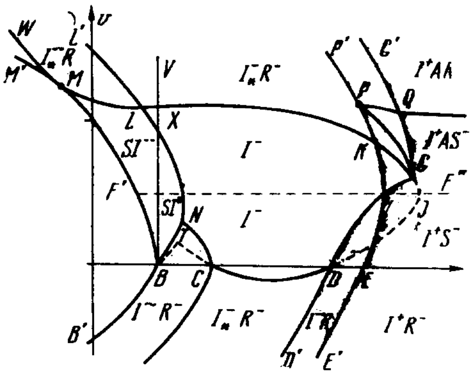

Fig. 1

Upstream of a slow subsonic wave ($u_1 < a_0$) the state of gas must be critical $\Phi(\rho_1, T_1) = 0$. This condition defines the intensity of the gas-dynamical shock wave spreading over the initial state in front of the ionizing wave. We note that in the case of arbitrary relationship between the dissipative coefficients the state of gas in front of a slow subsonic wave is arbitrary, while the variations of gas velocity components are bound to each other by a certain relationship.

For the convenience of subsequent exposition the velocity variation of the gas when it is traversed by a system of two waves, viz. one gas-dynamical shock wave, and one subsonic ionizing wave. This velocity variation belongs to area $VXNBMW$. At the same time to each point of area $LXBM$ correspond two ionizing waves propagating at different velocities. A single ionizing wave corresponds to the remaining points. The boundary of areas here considered consists of segments of curves along which one of the following conditions is satisfied.

1. The velocity variation in a slow ionizing wave is equal to that in a slow magnetohydrodynamic wave (curve BN).

2. The ionizing wave propagation velocity relative to the gas-dynamical shock wave is zero (curve NL).

3. Velocity $u_2 = a_{-2}$ is a Jouguet line (curve LMW).

4. The velocities of two ionizing waves corresponding to one and the same change of gas velocity coincide, which is represented by a bifurcation line (curve BM).

5. The ionizing wave velocity is zero (half-line BXV parallel to the v-axis).

We shall now consider the self-similar problem of gas motion generated by the motion of a piston. Let a flat perfectly conducting piston at instant of time $t = 0$ begin to move from position $x = 0$ at constant velocity $\mathbf{U} = u\mathbf{e}_x + v\mathbf{e}_y$. At the initial instant there is at rest in area $x > 0$ a nonconductive gas of constant density and pressure, and the electric field $\mathbf{E} = E\mathbf{e}_z$ and the magnetic field $\mathbf{H} = H_x\mathbf{e}_x + H_y\mathbf{e}_y$ are uniform. It is assumed that the piston motion generates an ionizing shock wave downstream of which the gas electrical conductivity is infinitely great. The gas motion is to be defined.

Boundary conditions at the piston surface follow from the stipulation for the electric field component tangent to the piston to vanish in a system of coordinates attached to the piston. If the conductive gas is in contact with the piston, this stipulation is reduced to the coincidence of the gas and piston velocity components. Normal velocity components also coincide by virtue of the impenetrability condition.

The problem of electrical discharge in which the gas motion is induced by a given constant electric field Ee_z maintained in plane x from instant $t = 0$, is a particular case of the piston problem stated above defined by condition $u = 0$, $v = cE/H_x$. The problem of discharge was considered in papers [4 and 5]. In particular, these papers had indicated the solution for a slow ionizing Jouguet wave followed by a rarefaction wave. In certain cases, however Jouguet waves which did not conform to the pattern shown on the diagram by the dotted line DJG were used for solving this problem. We note that [3 and 4] do not provide a complete analysis of the dependence of solution on the problem parameters.

The piston problem is also of interest because the solution of the self-similar problem of an arbitrary initial discontinuity decay may be considered as consisting of the piston problem solutions for each of the half-spaces (the function of the piston is in this case taken over by a constant discontinuity the velocity of which has to be determined).

The piston problem, as formulated above, is self-similar, and its solution must consist of surfaces of discontinuity and simple waves separated by areas in which all magnitudes have constant values. Sequence of waves is determined by their respective velocities. In this system of waves the electromagnetic wave comes first.

Changes of the electric and magnetic fields in an electro-magnetic wave are of the same magnitude. In the area behind the ionizing wave, where the gas conductivity is infinitely great $\mathbf{E} = \mathbf{H} \times \mathbf{v} / c$. In the area between the ionizing and magnetic waves the electric field transverse component is of the same order of magnitude. If the initial electric field is negligibly small as compared to the magnetic one, and $v/c \ll 1$, then changes of the magnetic field may be neglected in the electromagnetic wave, i.e. to consider the magnetic field in front of the ionizing wave as equal to the initial one. In the following we shall for simplicity's sake consider this case.

We shall represent the piston velocity $\mathbf{U} = u\mathbf{e}_x + v\mathbf{e}_y$ in plane uv by a point. Depending on the values of u and v the problem solution will be represented in the form of one, or another combination of waves following each other. The sets of points in the uv-plane to which correspond solutions consisting of identical combinations of waves form certain areas on that plane. The disposition and form of such areas were qualitatively analyzed for various initial values of the magnetic field, and of density and pressure. The disposition of these areas for the case of not very great p_1/H_x^2 and H_{y1}/H_x is shown on the diagram. Notations indicating wave combinations representing solutions of the problem for points of each area appear on the diagram inside of these areas. The following notations were used: I^+, $I^\sim$, I^- for fast, intermediate and slow supersonic ionizing waves; I^{--} for subsonic slow ionizing wave; I_*^- and I_*^{--} for slow supersonic and subsonic ionizing Jouguet waves, S^- and R^- for slow shock, and rarefaction magneto-hydrodynamic waves respectively; A for the Alfven discontinuity which rotates the magnetic field tangential component by 180°, and S for the gas-dynamical shock wave which transforms the initial state of the gas into the critical one.

A solution with a fast supersonic ionizing wave is possible when the piston velocity lies to the right of line $E'EFKPP'$. A fast ionizing wave may be followed by either slow shock waves, or slow rarefaction waves. Furthermore, solutions are possible in which an Alfven discontinuity which alters the sign of H_y spreads between the ionizing and the slow wave. Piston velocities lying above line FF'' correspond to solutions with an Alfven discontinuity (solutions in which H_y at the piston is zero correspond to points along line $F'FF''$).

If ionization occurs in an intermediate wave the solution of the problem may consist of a single intermediate wave, provided the piston velocity lies on segments BC, DE and PG. In areas adjacent to these segments the solution is represented by an intermediate ionizing wave followed by a slow magneto-hydrodynamic wave, either shock, or rarefaction, with solutions of the form $I^\sim S^-$, in area PGF, and $I^\sim R^-$ in area $P'PGG'$. We should point out that in the case represented on the diagram solutions with intermediate subsonic waves are absent.

A solution with slow supersonic ionizing waves is possible when the piston velocity belongs to area $G'QGFDD'C'CNLL'$. The solution may consist either of one ionizing wave (area $LKGFDCN$), or of a Jouguet ionizing wave (segments LKG and CD) followed by a slow rarefaction wave.

A solution with a slow subsonic ionizing wave is possible when the piston velocity belongs to area $VXNBMW$. In accordance with aforesaid a gas-dynamical shock wave which transforms the initial state of gas into the critical propagates in this case in front of the ionizing wave. The solution then either consists of two waves only, viz. one shock and one ionizing, or the ionizing wave is a Jouguet wave, in which case it can only be followed by a magneto-hydrodynamic rarefaction wave (area $M'MLL'$). We note that two different solutions with slow waves correspond to piston velocity values belonging to area $VXBMM'$. In this case there is in area $VXLL'$ one solution with a subsonic, and another with a supersonic ionizing wave, while in area $L'LXBMM'$ both solutions are with subsonic waves. These solutions coincide along line BMM'. Of these solutions the one containing the slower moving ionizing wave (the velocity of this wave vanishes along segment BXV) is apparently unstable. At piston velocities to the left of line $M'BMM'$ the solution is represented by an ordinary gas-dynamical wave downstream of which the gas nonconductive. We note that three different solutions with ionizing waves which may be either fast, intermediate, or slow correspond to piston velocities pertaining to area $P'PKFGQG'$.

Solutions possible in area FKG are: I^+AS^-, $I^\sim S^-$, and I^-; in area KPG: solutions I^+AS^-, $I^\sim S^-$, and $I_*^-R^-$; in area PGQ: solutions I^+AS^-, $I^\sim R^-$, and $I_*^-R^-$; in area $P'PQQ'$: solutions I^+AR^-, $I^\sim R^-$, and $I_*^-R^-$.

At the boundaries of area $P'PKFGQG'$ solutions coincide pairwise, viz. solution with an intermediate wave coincide along $FKPP'$ with solutions with a fast wave, and along $FGQG'$ with solutions with a slow ionizing wave. If any functions stipulating the solution of a problem, e.g., a change of the magnetic field in I^+A, $I^\sim$ and I^- as functions of u and v, then a three-valued part of that function will correspond to this area. For the same values of u and v a solution with a fast ionizing wave will correspond to a greater change of the magnetic field, while solutions with intermediate and slow waves will correspond to intermediate and slower changes respectively. In such cases the intermediate branch usually corresponds to unstable solutions.

The pattern presented above in the uv-plane applies to a perfect gas the initial parameter values of which are defined by the following system of inequalities:

$$\left|\frac{H_{y1}}{H_x}\right| < \left(\frac{\gamma+1}{\gamma-1}\right)^{1/2}\left(1-\frac{a_0}{a_{A1}}\right) \tag{1}$$

$$u^{*2} < a_{A1}^2 \frac{Z^2}{Z+(a_0/a_{A1})^2}\,\frac{4}{(\gamma+1)^2} \qquad \left(a_{A1}=\frac{H_x}{\sqrt{4\pi\rho_1}}\right) \tag{2}$$

Here u^* is the rate of change of the normal velocity component in the gas-dynamical wave which brings the gas to a critical state, ρ_1 is the initial density of gas, and Z is the smaller of the roots of Eq.

$$(\gamma-1)Z^2+\left[(\gamma-1)\frac{H_{y1}^2}{H_x^2}+\left(\frac{a_0^2}{a_{A1}^2}-1\right)(\gamma+1)\right]Z+(\gamma+1)\frac{a_0^2H_{y1}^2}{a_{A1}^2H_x^2}=0 \tag{3}$$

The energy expended on ionization has not been accounted for in relationships (1) and (2).

Similar patterns were obtained in the uv-plane for other initial values of pressure, density, and of the magnetic field. We shall not certain singularities of these.

Solutions with ionizing Jouguet waves in which H_y does not change its sign can only exist in those case in which inequalities (1) and (2) are simultaneously fulfilled, and in the latter the greater root of Eq. (3) is taken for Z. When condition

$$\frac{u^*}{a_{A1}} \geqslant 2\,\frac{1-a_0^2/a_{A1}^2}{\sqrt{(\gamma-1)(\gamma+1-2a_0^2/a_{A1}^2)}}$$

is satisfied, then there are no solutions with an intermediate wave in which the magnetic field remains unchanged, while solutions in which H_y does not change its sign contain either a fast supersonic, or a slow subsonic ionizing wave (the equality sign corresponds to the coincidence of points E and B on the diagram). Solutions with intermediate subsonic waves also occur for the same values of parameters, i.e. area $G'GFPP'$, or its part only lie to the left of line $L'LNB$. In the subsonic part of area $G'GFPP'$ the solution will then be two-valued, as in the limit case of high magnetic viscosity here considered a third solution with fast subsonic waves does not exist.

For sufficiently small values of H_x^2 / p_1 supersonic ionizing waves can only be fast. Subsonic ionizing waves (intermediate which change the magnetic field, and slow ones) do exist for arbitrarily small values of H_x, and their velocity tends to zero with H_x.

When $H_{y1} = 0$, then there are no solutions of the piston problem with either intermediate ionizing waves, or with Alfven and slow magneto-hydrodynamic shock waves. A change of sign of the y-component of velocity in the solution results in a change of sign H_y and of the y-component of velocity only.

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